Analysis of Decentralized AIMD-based EV Charging Control

Emin Ucer[1], Mithat C. Kisacikoglu[1], and Murat Yuksel[2]
[1]Dept. of Electrical and Computer Engineering, University of Alabama, Tuscaloosa, AL
[2]Dept. of Electrical and Computer Engineering, University of Central Florida, Orlando, FL
Emails: eucer@crimson.ua.edu, mkisacik@ua.edu, murat.yuksel@ucf.edu

***Abstract*—Additive Increase and Multiplicative Decrease (AIMD) control strategy has been studied for quite a long time. It was first introduced in 1989 and has been standardized as the Internet's congestion control algorithm. Its simple structure allows easy implementation and adaptation to the problems where fair and stable allocation of a limited resource among a group of agents/users is needed. From the power engineering perspective, this established method can be utilized for the control of electric vehicle (EV) charging, distributed energy sources (e.g. diesel machines, Photovoltaic panels (PV), and wind turbines), and energy storage units, as well as microgrid management and control. In this paper, we present the AIMD method and lay out a mathematical analysis on its stability and rate of convergence with a specific focus on electric vehicle charging. Later, we discuss necessary modifications needed to make the algorithm operate in a fully decentralized manner while still maintaining fairness. Finally, we demonstrate a case study where the algorithm is tested on EV charging control in the IEEE 37-bus distribution test feeder.**

***Index Terms*—Electric vehicles, AIMD, distributed control, EV grid integration.**

I. Introduction

The resource management is one of the core problems in many engineering disciplines. Distribution of the limited resources among multiple users while ensuring the system's safe and stable operation and a pragmatic fair allocation is a pressing challenge. Today's Internet owns its standards and protocols thanks to years of debates and research around this problem. Its early days suffered congestion control challenges as the number of endpoints drastically increased [1]. Observed congestion collapses [2], [3] revealed the need for a solution that assures both the stability of the system and the fair and efficient utilization of the network capacity. Because of the scale of the Internet, such a possible solution would be more suitable at the end-nodes in a decentralized design rather than a centralized control that has to keep track of every newly added end-point. To that end, the Additive Increase and Multiplicative Decrease (AIMD) algorithm [4] was proposed as a congestion avoidance solution at the end points. It is still now being used as the Internet's de-facto control method. By its very nature, the algorithm operates entirely with local measurements at the endpoints. It consists of two phases, which are additive increase (AI) and multiplicative decrease (MD). Transition between the phases is triggered by a congestion event. Every agent increases their network share linearly in the AI phase until a congestion event occurs in the network. In the event of congestion, the MD phase activates and each agent reduces their share by scaling it down geometrically and freeing up the resource for reallocation. This straightforward solution has been proved to be stable and maintain proportional fairness among agents [5].

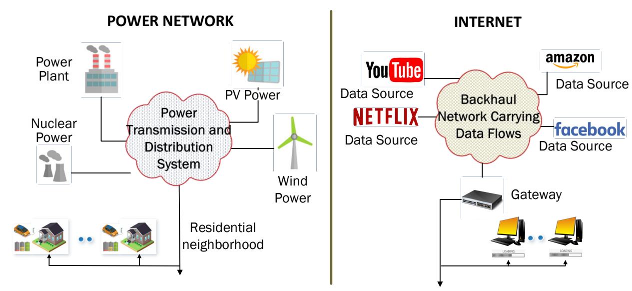

Figure 1. Illistrative analogy between power grid and the Internet.

Despite some differences, the problem of effectively sharing network capacity among greedy users while managing congestion in the network also arises when it comes to integrating large amounts of distributed energy sources and loads to the power distribution grid or microgrid. This phenomenon is illustrated in Fig. 1. Diesel generators, fuel cells, photovoltaic (PV) panels, and wind turbines are examples of such potential sources whereas electric vehicles (EVs) will constitute a significant part of the future loads. A smart grid can have several of these agents (either as a source or a load), each competing for the scarcity or the available capacity depending on whether they supply or consume power. The coordination and management of generation sources and the control of active loads in such a network with ever increasing end-points is a pressing challenge. It requires an additional communication network through which a central controller receives and sends commands. Even with such a costly investment, the system would still be highly vulnerable in terms of cyber security and stability.

The success of the AIMD inspired many in the power area to adapt the algorithm to power network problems. The EV charging problem has become one of the most suitable cases in which the AIMD approach finds an application area. Some early studies employed AIMD for EV charging and presented its performance analysis [6]–[8]. In [9], the authors enhanced the algorithm by taking some of the power system constraints into account. Undesired oscillations that might occur under the AIMD regulation and their causes are investigated in [10]. A comparison study between distributed AIMD-based and Price-Feedback based EV charging algorithms is presented in [11] by using an ideal centralized method as a benchmark. Most of these studies assume a simple binary communication link to inform agents of the grid congestion. However, as on the Internet, a fully autonomous operation based only on local measurements will be of a great value since it will significantly reduce the cost and complexity of the system.

In terms of an autonomous and decentralized designs, congestion decision can be made by evaluating the spare capacity in the grid. In [12], it is explained that local voltage can be used for this purpose. Furthermore, a congestion event occurs when the local voltage drops below a certain threshold value. Based on this fact, an AIMD EV charging algorithm using local voltage measurements is proposed in [13]. We presented an analysis in [14] to show how AIMD operates in terms of fairness and voltage violations once the voltage thresholds are properly set. In [15], we proposed a method on how to set these voltage thresholds dynamically by means of statistical analysis. A recent study [16] demonstrated that the local grid frequency can also be utilized in a decentralized AIMD algorithm to control grid-connected microgrids. Although these prior studies showed effectiveness of AIMD-based EV charging, an analytical study of how to set sevearl parameters of AIMD in this context is missing.

In this study, we first introduce the AIMD algorithm and present a mathematical modeling to help us analyze its dynamic behaviour. We give a solid proof on its stability and convergence signifying the roles and importance of its parameters. Later, we propose a method inspired by the TCP [17] protocol of the Internet to further enhance it with an autonomous feature based on local measurements. Finaly, we present a test case study where we implement the AIMD in EV charging control and discuss the results and improvements compared to uncontrolled case.

The rest of the paper is organized as follows: In Section II.A, we will introduce AIMD and present its convergence proof. In Section II.B, we will propose an EV charging algorithm and also present a method that makes the algorithm fully autonomous. In Section III, we will present the test setup used in this study and in Section IV, we will discuss the results and conclude in Section V.

II. Description and Analysis of the AIMD Algorithm

There are two aspects to the understanding of the AIMD algorithm. The first is the proper modeling of the algorithm that allows us to do the dynamic stability analysis, and the second is its congestion detection mechanism.

A. AIMD Modeling

The AIMD is a straightforward algorithm that has two operation phases. The additive increase (AI) phase takes part when there is available capacity in the system. In this phase, agents are allowed to increase their shares linearly by a rate $\alpha > 0$. In case of a congestion/capacity event, the algorithm switches to the multiplicative decrease (MD) phase where agents scale down their shares by a factor $0 < \beta < 1$. The capacity share of an agent at time $t+1$ can be formulated as follows:

$$w_i(t+1) = \begin{cases} w_i(t) + \alpha_i & \text{if there is no congestion} \\ w_i(t)\,\beta_i & \text{if congestion occurs} \end{cases} \tag{1}$$

where w_i denotes the share of the agent i. However this piece-wise formulation is not convenient for a dynamic analysis, and thus we need a proper mathematical model of the system. There are many approaches studied to model the algorithm. We use the switched system modeling approach presented in [18].

We assume that congestion events occur at discrete times, e.g. t_k, t_{k+1}. Then, the share of agent i at time $t > t_k$ can be described by a linear rule:

$$w_i(t) = \beta_i w_i(t_k) + \alpha_i (t - t_k); \quad t_k < t \le t_{k+1} \tag{2}$$

To obtain a discrete-time model, we can rewrite (2) by noting that $w_i(k)$ denotes the i^{th} agent's share at the k^{th} capacity event. Then, our model equation becomes:

$$w_i(k+1) = \beta_i w_i(k) + \alpha_i D(k) \tag{3}$$

where $D(k)$ is the time between two successive congestion events k and $k+1$ such that $D(k) = t_{k+1} - t_k$.

(3) represents a linear difference system. If we let $w_i(0)$ and d denote the initial share and the average time between two congestion events, respectively, and let k tend to a large number of n ($k \to n$), then it yields:

$$\begin{aligned} w_i(1) &= \beta_i w_i(0) + \alpha_i d \\ w_i(2) &= \beta_i(\beta_i w_i(0) + \alpha_i d) + \alpha_i d \\ &\vdots \\ w_i(n) &= \beta_i^n w_i(0) + \alpha_i d\,(\beta_i^{n-1} + \beta_i^{n-2} + \dots + \beta_i + 1) \end{aligned} \tag{4}$$

By using the geometric sum identity, we can modify (4) to obtain (5) such that:

$$w_i(n) = \beta_i^n w_i(0) + \alpha_i d \frac{1 - \beta_i^n}{1 - \beta_i} \tag{5}$$

(5) can be further simplified into:

$$w_i(n) = \beta_i^n \Big(w_i(0) - \frac{\alpha_i d}{1 - \beta_i}\Big) + \frac{\alpha_i d}{1 - \beta_i} \tag{6}$$

It is easy to see that (6) has a transient term represented by $\beta_i^n \, (w_i(0) - \frac{\alpha_i d}{1-\beta_i})$ and a steady-state term governed by $\frac{\alpha_i d}{1-\beta_i}$. Since β_i is always between 0 and 1, β_i^n term vanishes to zero as n tends to infinity and the system converges to (7):

$$\lim_{n \to \infty} w_i(n) = w_i^* = \frac{\alpha_i d}{1 - \beta_i} \tag{7}$$

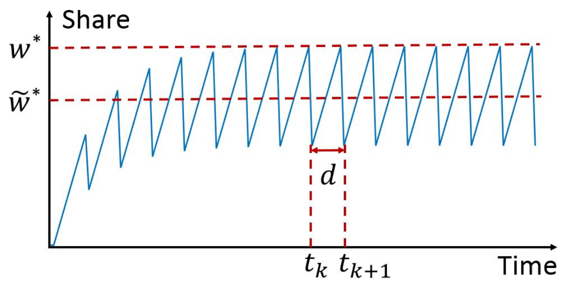

Figure 2. Capacity share $w(n)$ over time with AIMD in action as n tends to infinity.

We can also see that $_i{}^n$ provides an exponential convergence. The speed of this convergence depends on $_i$ and the difference between the initial and final shares $\{w_i(0) \quad w_i(1)\}$.

The average of the steady-state share value in (7) over two capacity events is donated by $\bar{w}$ and calculated as:

$$\begin{aligned} \bar{w}_i(k) &= \frac{1}{d} \int_{t_{k+1}}^{t_k} w_i(t) \quad dt \\ \bar{w}_i &= \ _i \frac{(1 + \ _i)}{2(1 \quad _i)} \quad d \end{aligned} \tag{8}$$

Fig. 2 demonstrates a typical share waveform under AIMD with constant , and d parameters.

B. Congestion Detection

(7) shows us that the final share of an agent depends on the algorithm parameters , and the period of the congestion detection d. If the parameters are set to be the same across the network and each agent is notified of the congestion simultaneously (i.e., centralized control), then this equation guarantees that every agent will get the same share from the network, establishing an ideal fairness. For a fully autonomous control, however, congestion has to be detected by each agent using local information. In the Internet, this is done by measurements at every round trip time (RTT). When data packets sent from one end-point arrive at their destination, an acknowledgement (ack) is sent back to inform the sender of its packet delivery. Thus, RTT is the time it takes between sending the packets and receiving the ack.

A congestion in the network increases the packet queues at routers and thus results in longer RTTs. This clearly shows a correlation between RTTs and the congestion level. Therefore the Internet's TCP/IP protocol uses RTT to autonomously detect the congestion by comparing it with a timeout value. This value is called the re-transmission timeout (RTO) and it is calculated on-the-fly by using the statistics of the measured RTTs. Some agents might detect the congestion more often than others depending on their closeness to the congestion's location. This naturally results in the average time d, and thus w_i, being slightly different from one agent to another and creates a proportional fairness among agents. The agents whose packets are traversing longer paths (i.e., using more link capacity in total) get penalized accordingly and converge to a lower $!_i$.

Algorithm 1 AIMD algorithm for EV charging network

Input: Previous charging current: $I_i(t)$
Output: New charging current: $I_i(t+1)$
Parameter: Increase parameter: $(t) > 0$
Parameter: Decrease parameter: $0 < \ (t) < 1$
if $V(t) > V_{th}$ **and** $V(t) > V_{min}$ **then**
    $I_i(t+1) = I_i(t) + \ _i$
else
    $I_i(t+1) = \ _i \quad I_i(t)$
end if

In a power distribution network, an autonomous congestion detection mechanism can be realized by using local voltage measurements [12], [15]. For EV charging control, the AC charging current $I_i(t)$ can be chosen as the share parameter $w_i(t)$, and the algorithm can be implemented as described in Algorithm 1. The detection condition checks whether the measured node voltage $V(t)$ is higher than the threshold voltage V_{th} and the minimum allowable grid utilization voltage V_{min}. If it is less than either of these, MD phase kicks in, and the charging current is reduced by $_i$. Just like RTO in the Internet, the key parameter V_{th} also has to be locally calculated for each agent by means of voltage measurement statistics. In essence, V_{th} can be calculated as an outlier of the estimated average voltage value based on *Chebyshev's outlier estimation* [19]. Chebyshevs Inequality states for any random variable X with mean and variance ^{2} that

$$P(X \not 2 j \quad k \ j) \quad \frac{1}{k^2} \tag{9}$$

This means that $100 \quad (1 \quad \frac{1}{k^2})\%$ of the measured X values are to be between k and $+ k$. In TCP congestion control, the recommended value for k is 4. For our study, we too chose k to be 4, which means that around 93% of the time the true average of voltage must be within the measured voltages. Then, the 7% should correspond to the outlier, which can be used as V_{th} as follows:

$$V_{th}(t+1) = V(t+1) \quad 4 \ \hat{V}(t+1) \tag{10}$$

where $V(t+1)$ and $\hat{V}(t+1)$ correspond to the mean and standard deviation of the measured voltage. Using exponentially weighted moving average (EWMA), we estimate V and $\hat{V}$ values, then:

$$\begin{aligned} V(t+1) &= \quad V(t) + (1 \quad) \ V(t) \\ \hat{V}(t+1) &= ! \quad jV(t) \quad V(t)j + (1 \quad !) \ \hat{V}(t) \end{aligned} \tag{11}$$

where and $!$ are the coefficients that determine the contribution of the recent measurements to the average values and thereby, the response time of the system. For this study, we chose $= 1$, $= 0{:}5$, $= 0{:}7$, and $! = 0{:}2$. While and are the default values in TCP congestion control, we chose the latter two parameters driven by TCP as well.

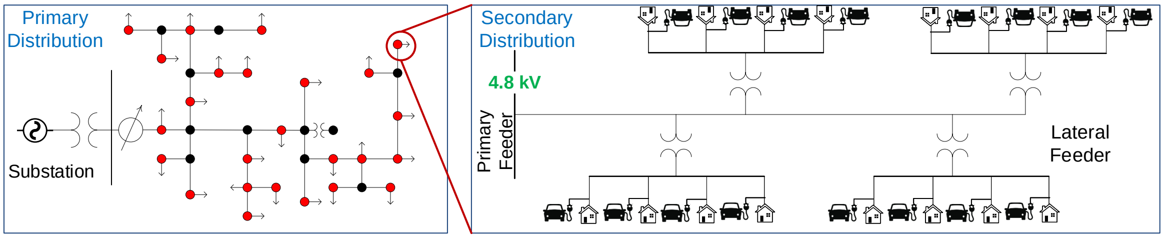

Figure 3. Primary and secondary distribution network implemented in the MATLAB model.

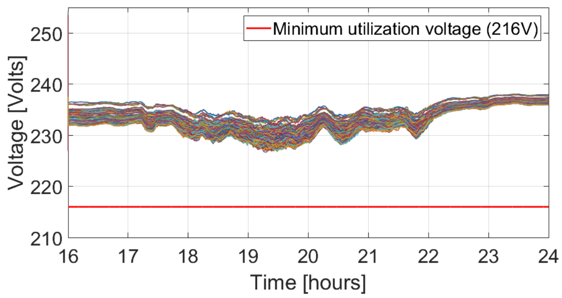

Figure 4. Voltage profiles of 416 households for %0 EV penetration.

III. Simulation Setup

For testing the proposed algorithm, the distribution grid model in [20] is used. The modeled grid is a 2.5 MVA, 4.8 kV, 37-bus, three-phase balanced network implemented in MATLAB Simulink. Each node is designed as a neighborhood with 16 houses, making a total of 416 customers. EV charging loads are incorporated in the system in parallel with conventional residential loads as shown in Fig 3. EV loads are modeled such that they have a rated capacity of 60 kWh and maximum charging current of 30 A. For this study, all EVs are assumed to arrive after 16:00 with a state of charge (SOC) higher than 80%. Residential power consumption are generated uniquely for each household from a consumption probability distribution function (PDF) based on 16 days of consumption data downloaded from E-gauge [15], [21]. Three EV penetration levels (25%, 50% and 100%) will be tested under AIMD. The results will be compared with the 0% and 100% penetration cases with uncontrolled charging.

IV. Results and Discussion

In order to see the impact of the proposed algorithm and make a better comparison, we consider two extreme cases. One such a case is where there is no EV penetration (baseline load) and the other is 100% EV penetration. All EVs are charged at full power (7 kW).

Figs. 4 and 5 show the root mean square (rms) voltage waveforms of all 416 households between 16:00-24:00 for 0% and 100% EV penetrations (no charging control), respectively. As shown, grid voltages considerably drop compared to the base load case. Without a controlled charging at 100% penetration, the voltages drop even below the minimum allowable voltage limit (216 V) during peak-hours (18:00-20:00).

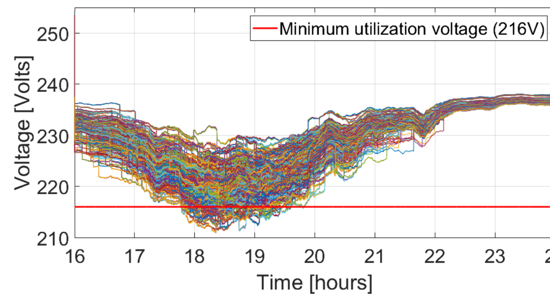

Figure 5. Voltage profiles of 416 households for 100% EV penetration with charging at rated power of 7 kW.

The voltage waveforms of 416 households at 100% EV penetration with AIMD control is presented in Fig. 6. This figure shows that with AIMD in action, the voltages are successfully held above the critical level at 100% EV penetration avoiding any voltage violation.

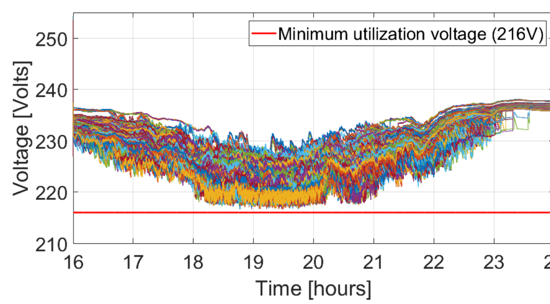

Figure 6. Voltage profiles of 416 households for 100% EV penetration with AIMD charging.

The total power of the system between 16:00-24:00 for different penetration levels along with the base load (0% EV) is given in Fig. 7. This shows that the peak-load of the grid (purple) is successfully shifted towards off-peak hours (yellow) with fully autonomous control. SOC variations of the vehicles for 100% penetration level with the AIMD control is given in Fig. 8. This figure shows that the EVs managed to get fully charged by midnight by shifting the peak-load and avoiding voltage violations.

The average charging power of each EV in the ascending order of distance to substation is also presented in Fig. 9. This figure shows that a proportional fairness has been established around 3-4 kW for all penetration levels in the grid. This means the customers closer to the substation benefit higher